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XXXVI. *Some New Theorems for computing the Areas of certain Curve Lines: By Mr. John Landen, F.R.S.*

Read June 28, 1770. **T**HE learned editor of Mr. Cotes's *Harmonia Mensurarum* first gave us, in that book, the celebrated theorems for computing the areas of the curves whose ordinates are expressed by $\frac{x^p}{a^n + x^n}$, $\frac{x^p}{a^n + x^n \times e^n + x^n}$, or $\frac{x^p}{a^{2n} + 2ca^n x^n + x^{2n}}$; and several other writers have since done the like. Which theorems consist of many terms, being obtained by previously resolving the expression for the ordinate, into others of a more simple form. Now I have found, that the *whole* area of every such curve (when finite) may be assigned by theorems remarkably concise, without the trouble of resolving the expression for the ordinate as aforesaid: and, as in the resolution of problems, the whole area of a curve is more commonly wanted than a part thereof; and as these new theorems enable us to compute such whole areas as above-mentioned, or the whole fluents of $\frac{x^p \dot{x}}{a^n + x^n}$, $\frac{x^p \dot{x}}{a^n + x^n \times e^n + x^n}$, and $\frac{x^p \dot{x}}{a^{2n} + 2ca^n x^n + x^{2n}}$, with

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admirable facility ; I do myself the honour of communicating them to the Royal Society, presuming they may be thought worthy to be published in the Philosophical Transactions.

THEOREM I.

m being any positive integer or fraction, and n any such integer or fraction, greater than m ; the *whole* area of the curve, whose abscissa is x , and ordinate

$$\frac{x^{m-1}}{a^n + x^n} \text{ is } = \frac{a^{m-n}}{fn} \times A.$$

THEOREM II.

m and n being as before-mentioned, the *whole* area of the curve, whose abscissa is x , and ordinate

$$\frac{x^{n+m-1}}{a^n + x^n \times e^n + x^{2n}} \text{ is } = \pm \frac{a^{n+m}-e^{n+m}}{a^n - e^n} \times \frac{A}{fn}.$$

Note. When e is $=a$, the expression for the area becomes $= \frac{ma^{n+m-n}}{fn^2} \times A.$

THEOREM III.

m and n being as in the preceding theorems, the *whole* area of the curve, whose abscissa is x , and

$$\text{ordinate } \frac{x^{n+m-1}}{a^{2n} + 2ca^n x^n + x^{2n}} \text{ is } = \frac{ga^{n+m-n}}{bfn} \times A.$$

Note. If m be $=0$, the area will be $= \frac{a^{-n}B}{bn}.$

In these theorems,

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A denotes the femi-periphery of the circle, whose radius is 1 ;

B an arc of the same circle, whose cosine is c , and sine b ;

f the sine of the arc $\frac{m}{n} \times A$;

g the sine of the arc $\frac{m}{n} \times B$.

Concerning the investigation of these theorems, it is sufficient to say, they are directly obtained by the help of my new method of comparing curvilinear areas, inserted in the *Philos. Transact.* for the year 1768.

It is obvious, that, by means of the above theorems, we may very readily compute the *whole* areas (when finite) of the curves, whose ordinates are

$\frac{x^p}{p + qx^n + rx^{2n} + x^{3n}}$, and $\frac{x^p}{p + qx^n + rx^{2n} + sx^{3n} + x^{4n}}$, &c. seeing these expressions may be easily transformed into others similar to those already considered.